

1st Order Modeling of a SAW Delay Line using MathCAD®

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ABSTRACT

To aid in the development of SAW sensors for Integrated Vehicle Health Monitoring applications, a first order model of a SAW Delay line has been created using MathCad®. The model implements the Impulse Response method to calculate the frequency response, impedance, and insertion loss. This paper presents the model and the results from the model for a SAW delay line design.

1. INTRODUCTION

Integrated Vehicle Health Monitoring (IVHM) of aerospace vehicles requires rugged sensors having reduced volume, mass, and power that can be used to measure a variety of phenomena. Wireless systems are preferred when retro-fitting sensors onto existing vehicles [1]. Surface Acoustic Wave (SAW) devices are capable of sensing: temperature, pressure, strain, chemical species, mass loading, acceleration, and shear stress. SAW technology is low cost, rugged, lightweight, and extremely low power. Passive wireless sensors have been developed using SAW technology. For these reasons new SAW sensors are being investigated for aerospace applications.

2. BACKGROUND

The SAW Delay line structure is composed of two sets of InterDigitated (IDT) metal fingers on a piezoelectric substrate (Figure 1). This configuration can be used as a delay line or as bandpass filter [2]. Sensors can be developed by adding a sensing medium between the two IDTs. When one of the IDTs is excited by a sinusoidal electrical signal the piezoelectric effect generates mechanical waves. The waves that propagate to the other IDT generate an electrical signal through the inverse piezoelectric effect. SAW devices can be designed to minimize dispersion, attenuation, or diffraction [3].

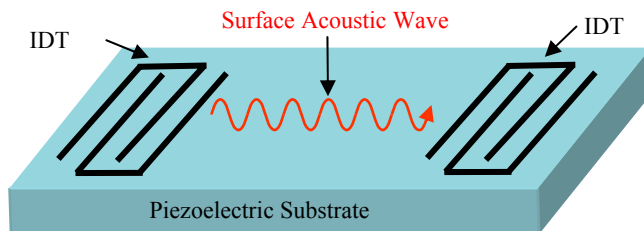


Figure 1. Basic SAW Delay line Device.

Previously, the Impulse Response method was implemented using the Very High Speed IC Hardware Design Language (VHDL) [4] but only in the time domain.

Attempts to implement the model using frequency domain analysis of VHDL-AMS were not successful. A reason may be the lack of support for frequency domain analysis. Although frequency domain analysis exists in the language it is rudimentary and is not sufficient for some general frequency domain modeling [5]. The mathematical approach performs frequency domain calculations within MathCad. MathCad is a mathematical modeling package and therefore does not exhibit any of the limitations of VHDL or VHDL-AMS.

3. MODEL IMPLEMENTATION

The Impulse Response method developed by Hartmann, Bell and Rosenfeld [6] was used to model the SAW delay line. This is a first order model that does not take into account any second order effects such as reflections, however, it does model the piezoelectric, mechanical and electrical behaviors of the SAW device. This model is only valid for transducers where the IDTs are un-weighted (un-apodized). The model assumes that the finger overlap or aperture is constant and that the metallization ratio between the fingers and spaces is 0.5. This model calculates the frequency response, conductance, susceptance, impedance, and other electrical parameters. The Impulse Response method uses the Mason equivalent circuit shown in Figure 2. In the circuit $G_a(f)$ is the radiation conductance, $B_a(f)$ is the acoustic susceptance, and C_T is the total capacitance.

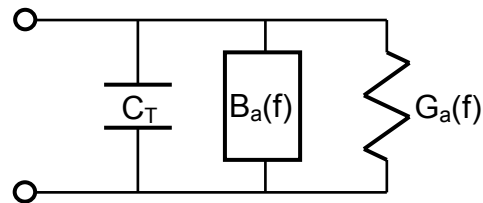


Figure 2. Mason equivalent circuit model.

From the Impulse Response model one can calculate wavelength (λ), and the number of finger pairs (N_p) using the following equations:

$$\lambda = \frac{v}{f_0} \quad (1)$$

$$N_p = \text{round}\left(\frac{2}{NBW} f_0\right) \quad (2)$$

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where v is the acoustic velocity in the media, f_0 is the center or synchronous frequency, and NBW is the null bandwidth or fractional frequency.

3.1 Frequency Response

The frequency response of a SAW system can be implemented using the Impulse Response method. For a single IDT, the frequency response is shaped by the sinc function, and is calculated by the following equation [6]:

$$|H(f)| = 2k\sqrt{(C_s f_0)} N_p \frac{\sin(X)}{X} \quad (3)$$

where f is the frequency, k is the piezoelectric coupling coefficient, C_s is the capacitance for a finger pair per unit length. The variable X is used to simplify the equation and is defined as [6]

$$X = N_p \pi \frac{(f - f_0)}{f_0} \quad (4)$$

When two IDTs are used as in the case of a SAW delay line, the frequency response for the system is found by combining the frequency response for each IDT [6]:

$$H_T(f) \cong H_1(f) \cdot H_2(f) \quad (5)$$

Often the frequency response is normalized using the log equation:

$$H_n(f) = 20 \log \left(\frac{H_1(f) H_2(f)}{H_1(f_0) H_2(f_0)} \right) \quad (6)$$

3.2 Radiation Conductance

The real part of the input admittance is called the radiation conductance. The radiation conductance is also shaped by the sinc function [6] and is found by

$$G_a(f) = 8k^2 C_s W_a f_0 N_p^2 \left| \frac{\sin(X)}{X} \right|^2 \quad (7)$$

where W_a is the aperture or overlap height of the fingers. The radiation conductance is normalized by using the following equation:

$$G_n(f) = \frac{G_a(f)}{G_a(f_0)} \quad (8)$$

3.3 Acoustic Susceptance

The third element of the model is the imaginary part of the input admittance which is also called the acoustic

susceptance. The acoustic susceptance is the acoustic wave phenomena modeled as an electrical parameter. The acoustic susceptance is found by taking the Hilbert transform of the radiation conductance [6] and is given by:

$$B_a(f) = \frac{G_a(f_0) \sin(2X) - 2X}{2X^2} \quad (9)$$

The acoustic susceptance is normalized by using the following equation:

$$B_n(f) = \frac{B_a(f)}{G_a(f_0)} \quad (10)$$

Notice that the acoustic susceptance is normalized using the radiation conductance since the acoustic susceptance at the synchronous frequency is zero.

3.4 Admittance and Impedance

The total admittance [7] is found by combining the radiation conductance, the acoustic susceptance and the total capacitance. The total admittance is given by

$$Y = G_a + j(2\pi f C_T + B_a) \quad (11)$$

The total static capacitance (C_T) for the IDT is found by multiplying the capacitance per unit length for a pair of fingers (C_s) times the finger overlap or aperture (W_a) times the number of finger pairs (N_p).

$$C_T = N_p C_s W_a \quad (12)$$

If equation (11) is inverted it will yield the impedance of the system [7]:

$$Z(f) = \frac{1}{(G_a + j(2\pi f C_T + B_a(f)))} \quad (13)$$

3.5 Insertion Loss

It is very useful to calculate the insertion loss of electrical systems. For SAW devices the insertion loss is a function of frequency, and is calculated using conductance, susceptance, and the load resistance R_g [8]:

$$IL(f) = -10 \log \left[\frac{2G_a(f)R_g}{(1 + G_a(f)R_g)^2 + [R_g(2\pi f C_T + B_a(f))]^2} \right] \quad (14)$$

The minimum insertion loss occurs when $f = f_0$ the synchronous frequency.

3.6 Aperture Optimization

An optimal design must match the IDT resistance (real impedance) to the input resistance. The device aperture (W_a) is often adjusted so that the IDT design achieves the correct IDT resistance. The following equation was used to optimize the aperture in terms of the input resistance:

$$W_a = \frac{1}{R_{in}} \left(\frac{1}{2f_0 C_s N_p} \right) \frac{(4k^2 N_p)}{(4k^2 N_p)^2 + \pi^2} \quad (15)$$

where R_{in} is the input resistance.

3.7 Matching Network

If impedance matching is required, then a series inductor can be used to cancel out the static capacitance at the synchronous frequency. To calculate the series inductor value use the following [9]:

$$Inductor = \frac{1}{(2\pi f_0)^2 C_T} \quad (16)$$

4. SAW DELAY LINE EXAMPLE

A simple SAW delay line that consists of two identical IDTs will be used to demonstrate the model. The synchronous frequency is 52.633 MHz. The substrate is ST cut Quartz. The Null BandWidth (NBW) is 1.5 MHz. The delay length between the two IDTs is 5 wavelengths. Both the load and input resistance are assumed to be 50 Ω . The selection of a substrate material determines the capacitance, $C_s = 0.503385$ pf/cm, the piezoelectric coefficient, $k = 0.04$, and the acoustic velocity, $v = 3158$ m/s for the SAW device [10]. Using these values in equation (15) yields an optimized aperture of 2399.0 μm .

5. ANALYSIS AND LAYOUT

The plots found in Appendix A (MathCad file), present the frequency response, radiation conductance, acoustic susceptance, and the insertion loss of the example design. In addition to the plots, several parameters are calculated and written into a summary report to aid in analyzing the design. As shown in Figure 3 the width of the fingers is 15 μm , the number of fingers is 70, aperture height is 2399.0 μm , there is a 5 wavelength delay, and the bus bar height is 500 μm . The first eight parameters are required for automatic layout generation. The aperture height is the second parameter that is optimized. The aperture width is optimized to match 50 Ω , since most test equipment has input and output impedances of 50 Ω [11]. Manual intervention is necessary during the analysis phase to check over the plots, however, future improvements will incorporate automatic checking of the parameters, and automatic optimization tools that would further reduce the amount of human interaction that is needed.

<<< SAW Delay line Summary Report >>>	
Finger Height	$W_t = 2.499\text{E-}003$ m
Aperture Width	$W_a = 2.399\text{E-}003$ m
Finger Width/Spacing	$E_w = 15.000\text{E-}006$ m
Number of Finger Pairs	$N_p = 70.000\text{E+}000$
Bus Bar Height	$BBH = 500.000\text{E-}006$ m
Delay Length (wavelengths)	Delay = 5.000E+000
X Offset	$X = 0.000\text{E+}000$
Y Offset	$Y = 0.000\text{E+}000$
Wavelength	$\lambda = 60.000\text{E-}006$ m
Synchronous Frequency	$f_0 = 52.633\text{E+}000$ MHz
Acoustic Velocity	$v = 3.158\text{E+}003 \frac{\text{m}}{\text{s}}$
Series Matching Inductor	Inductor = 1.082E-006 H
Minimum Insertion Loss (@f0)	$IL(f_0) = 14.247\text{E+}000$
Total Capacitance (Single IDT)	$C_T = 8.452\text{E-}012$ F

Figure 3. Summary report the SAW delay line parameters.

A number of assumptions must be made before the parameters that have been calculated can be used to realize a layout. These values are the height of the bus bars on the top and bottom of the fingers, the length of the fingers that are not overlapped (not part of the aperture), and the X and Y offset values. In the earlier example the following parameters are used; a bus bar height of 500 μm , and un-overlapped finger length of 100 μm , and for simplicity an offset of (0.0, 0.0). Using the values from Figure 3, an automatic layout can be performed to create the layout and netlist of the device [12]. The netlist has been used to fabricate the device (Figure 4).

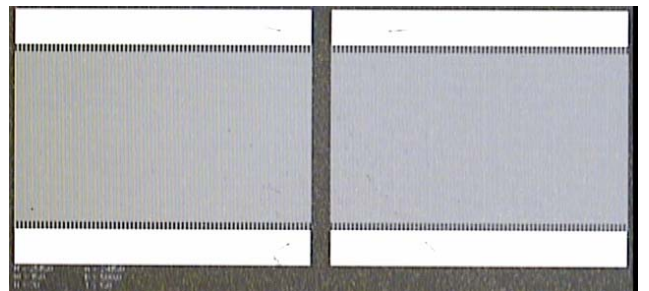


Figure 4. Surface Acoustic Wave Delay Line Device.

The calculated and the measured insertion loss of a SAW delay line is given in Figure 5. Although the peak loss of the measured device is larger than was calculated, the side-lobe rejection is better than was expected. Note that the calculated values are first order only.

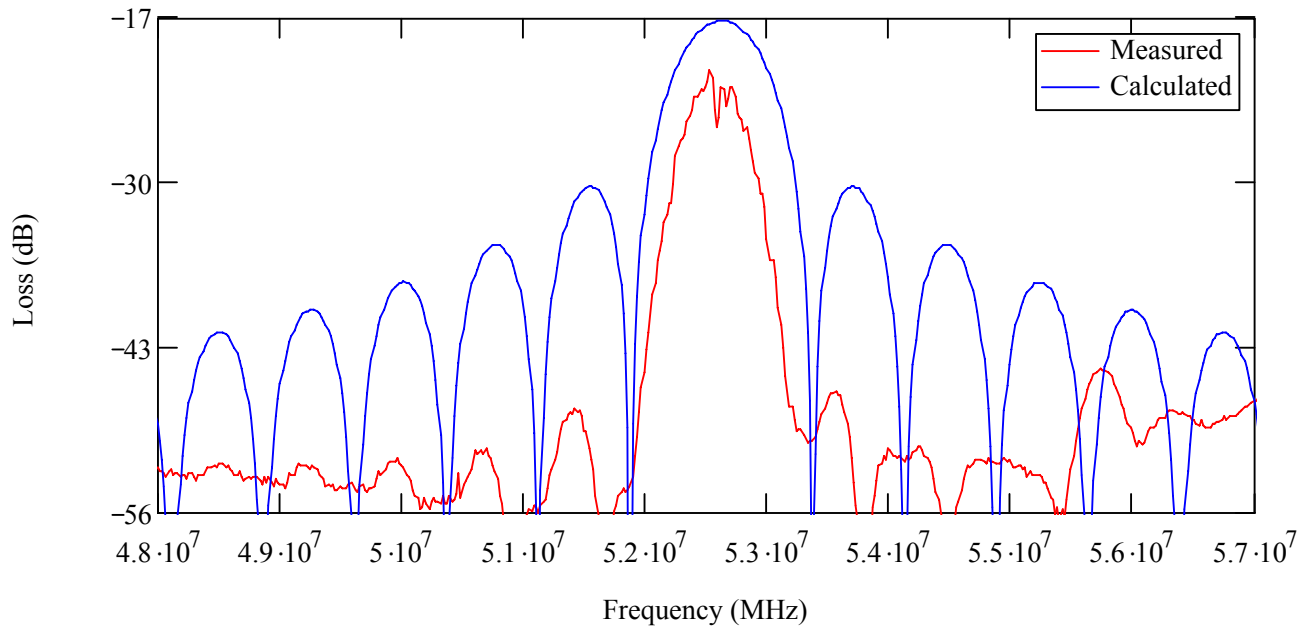


Figure 5. Calculated and Measured Results of a SAW Delay Line.

6. CONCLUSION

An implementation of the Impulse Response Model for a SAW delay line device using MathCad has been presented. While VHDL and VHDL-AMS are typically used for modeling SAW devices, mathematical tools such as MathCad do not have the same limitations and therefore are better suited for implementing models in the frequency domain. The MathCad model has been presented, along with the results from an example SAW delay line. Some rudimentary optimization was performed and a report was generated with all of the salient design parameters, and automatically generated a layout of the example design. Future work will include second order effects such as reflections, triple transit echoes, and temperature effects. This work will use transmission line theory to implement SAW devices.

7. REFERENCES

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8. Appendix A (MathCad File)

Constants:

$$\mu := 10^{-6} \quad \eta := 10^{-9} \quad p := 10^{-12} \quad j := \sqrt{-1}$$

Inputs:

$$f_0 := 52.633333 \cdot \text{MHz}$$

$$\text{Substrate} := \text{"ST Cut Quartz"}$$

$$\text{NBW} := 1.5 \cdot \text{MHz}$$

$$\text{Delay} := 5.0$$

$$X := 0 \quad Y := 0$$

$$W_f := 100 \cdot \mu\text{m}$$

Assumptions:

$$v := 3158 \cdot \frac{\text{m}}{\text{s}}$$

$$k := \sqrt{0.0016}$$

$$\text{BBH} := 500 \cdot \mu\text{m}$$

$$R_g := 50 \cdot \Omega$$

$$C_s := 0.503385 \times 10^{-12} \cdot \frac{\text{F}}{\text{cm}}$$

Synchronous Frequency

Substrate Material

Null Bandwidth

Delay Length in Wavelengths

X and Y offsets for SAW device

Finger to Bus Bar Spacing

Acoustic Velocity for ST Quartz

Piezoelectric Coefficient

Bus Bar Height.

Electrical Impedance is 50Ω .

Capacitance/ finger pair/ m

Calculations:

$$\lambda := \frac{v}{f_0}$$

$$\lambda = 60 \mu\text{m}$$

Wavelength

$$E_w := \frac{\lambda}{4}$$

$$E_w = 15 \mu\text{m}$$

Electrode Width:

$$N_p := \text{round}\left(\frac{2}{\text{NBW}} \cdot f_0\right)$$

$$N_p = 70.000000\text{E}+000$$

Number of Finger pairs

$$W_a := \frac{1}{50 \cdot \Omega} \cdot \left(\frac{1}{f_0^2 \cdot C_s \cdot N_p}\right) \cdot \text{Re}\left(\frac{1}{4 \cdot k^2 \cdot N_p + j \cdot \pi}\right)$$

$$W_a = 2399 \mu\text{m}$$

Finger Aperture

$$W_t := W_a + W_f$$

$$W_t = 2.499 \times 10^3 \mu\text{m}$$

Finger Height

$$x(f) := \text{if}\left[f = f_0, 0, N_p \cdot \pi \cdot \left(\frac{f - f_0}{f_0}\right)\right]$$

$$G_a(f) := 8 \cdot k^2 \cdot C_s \cdot W_a \cdot f_0 \cdot N_p^2 \cdot \left(\frac{\sin(x(f))}{x(f)}\right)^2$$

Radiation Conductance

$$C_T := N_p \cdot C_s \cdot W_a$$

Total IDT Capacitance

$$B_a(f) := G_a(f_0) \cdot \frac{(\sin(2 \cdot x(f))) - 2 \cdot x(f)}{2 \cdot x(f)^2}$$

Acoustic Susceptance

$$H(f) := 2 \cdot k \cdot \sqrt{C_s \cdot f_0} \cdot N_p \cdot \frac{\sin(x(f))}{x(f)}$$

Frequency Response

$$Y_T(f) := \left[G_a(f) + j \cdot (2 \cdot \pi \cdot f \cdot C_T + B_a(f)) \right]$$

Admittance

$$Z_T(f) := \frac{1}{Y_T(f)} \quad Z_T(f_0) = (50.000 - 350.624j) \Omega$$

Impedance

$$IL(f) := -10 \log \left[\frac{2 \cdot G_a(f) \cdot R_g}{(1.0 + G_a(f) \cdot R_g)^2 + [R_g \cdot (2 \cdot \pi \cdot f \cdot C_T + B_a(f))]^2} \right]$$

Insertion Loss

$$\text{Inductor} := \frac{1}{(2 \cdot \pi \cdot f_0)^2 \cdot C_T}$$

Matching Series Inductor

Values for Plotting:

$$f_{\min} := 0.5 \cdot f_0 \quad f_{\min} = 26.317 \text{ MHz} \quad f_{\max} := 1.5 \cdot f_0 \quad f_{\max} = 78.950 \text{ MHz} \quad df := \frac{f_{\max} - f_{\min}}{10000}$$

$$f := f_{\min} + df \cdot i \quad H_n(f) := 20 \cdot \log \left(\left| \frac{H(f)}{H(f_0)} \right| \right) \quad G_n(f) := \frac{G_a(f)}{G_a(f_0)} \quad B_n(f) := \frac{B_a(f)}{B_a(f_0)}$$

